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Observations on the paper entitled "On Finding Integer Solutions to Sextic Equation with Three Unknowns $x^2+y^2=8z^6$"

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Abstract

In this paper ,we obtain many non-zero distinct integer triples x, y, z satisfying the non-homogeneous ternary sextic equation given by $x^2+y^2=8z^6$.

Keywords : ternary sextic ,non-homogeneous sextic ,integer solutions

Introduction

It is worth to observe that higher degree Diophantine equations with multiple variables are rich in variety. While attempting to collect sixth degree Diophantine equations with three unknowns ,the authors came across the paper [1] entitled

"On finding Integer Solutions to Sextic Equation with Three Unknowns $x^2+y^2=8z^6$". In the above paper ,the authors have presented a few choices of integer solutions. Albeit tacitly ,there are other choices of integer solutions to the considered equation which is the main aim of this paper.

Method of analysis

The non-homogeneous ternary sextic diophantine equation to be solved is

$$x^2+y^2=8z^6 \quad (1)$$

Introduction of the linear transformations

$$x=2X, y=2Y \quad (2)$$

in (1) leads to

$$X^2 + Y^2 = 2z^6 \tag{3}$$

The process of obtaining different sets of integer solutions to (1) is illustrated below :

Set 1 :

Assume

$$z = a^2 + b^2 \tag{4}$$

Express the integer 2 on the R.H.S. of (3) as

$$2 = (1+i)(1-i) \tag{5}$$

Substituting (4) & (5) in (3) and employing the method of factorization,

define

$$X + iY = (1+i)(a+ib)^6 \tag{6}$$

Equating the real and imaginary parts in (6) ,the values of X, Y are obtained.

In view of (2) , one has

$$x = 2[f(a,b) - g(a,b)], y = 2[f(a,b) + g(a,b)] \tag{7}$$

where

$$f(a,b) = (a^6 - 15a^4b^2 + 15a^2b^4 - b^6)$$
$$g(a,b) = (6a^5b - 20a^3b^3 + 6ab^5)$$

Thus ,(4) and (7) represent the integer solutions to (1).

Set 2 :

Write (3) as

$$X^2 + Y^2 = 2z^6 * 1 \tag{8}$$

Express the integer 1 on the R.H.S. of (8) as

$$1=\frac{(p^2-q^2+i2pq)(p^2-q^2-i2pq)}{(p^2+q^2)^2} \tag{9}$$

Substituting (4),(5) & (9) in (8) and employing the method of factorization,

define

$$X+iY=\frac{(1+i)(p^2-q^2+i2pq)(f(a,b)+ig(a,b))}{(p^2+q^2)} \tag{10}$$

Equating the real and imaginary parts in (10) and replacing a by $(p^2+q^2)A$,

b by $(p^2+q^2)B$,the corresponding integer solutions to (1) are given by

$$x=2(p^2+q^2)^5[(p^2-q^2-2pq)f(A,B)-(p^2-q^2+2pq)g(A,B)],$$
$$y=2(p^2+q^2)^5[(p^2-q^2-2pq)g(A,B)+(p^2-q^2+2pq)f(A,B)],$$
$$z=(p^2+q^2)^2(A^2+B^2)$$

Note :

The integer 2 on the R.H.S. of (3) may also be expressed as the product of complex conjugates as below:

$$2=\frac{[(p^2-q^2-2pq)+i(p^2-q^2+2pq)]\,[(p^2-q^2-2pq)-i(p^2-q^2+2pq)]}{(p^2+q^2)^2}$$

Repeating the analysis as in Set 1 and Set 2 ,two more sets of integer solutions to (1) are found.

Set 3 :

Introduction of the transformations

$$x=2(2rs+r^2-s^2),y=2(2rs-r^2+s^2) \tag{11}$$

in (1) leads to the equation

$$r^2 + s^2 = z^3 \tag{12}$$

There are two sets of solutions to (12) which are represented by

$$r = m(m^2 + n^2)\ ,s = n(m^2 + n^2)\ ,z = (m^2 + n^2)$$

and

$$r = m(m^2 - 3mn^2)\ ,s = n(3m^2 - n^2)\ ,z = (m^2 + n^2)$$

In view of (11) , one obtains two more sets of integer solutions to (1).

Conclusion :

In this paper ,we have presented sets of integer solutions to the non-homogeneous sextic equation with three unknowns given in title that are different from [1].As Diophantine equations are rich in variety , one may search for other choices of sextic equations with multiple variables for determining their respective integer solutions.

Reference

[1] . J.Shanthi , S.Vidhyalakshmi , M.A.Gopalan , On Finding Integer Solutions to Sextic Equation with Three Unknowns $x^2 + y^2 = 8z^6$,The Ciencia and Engengaria-Science and Engineering Journal ,11(1) ,350-355 ,2023.